

# Relic gravitons from non-singular string cosmologies

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**Abstract.** In the context of the pre-Big Bang scenario of string cosmology, we propose a modified equation for the evolution of the tensor perturbations, which includes the full contribution of possible higher-order curvature and coupling corrections required to regularise the background evolution. We then discuss the high-frequency branch of the spectrum of primordial gravitons. Preliminary results suggest that the slope of the string branch of the spectrum is too steep to allow for the cosmological background of relic gravitons to match the frequency range probed by planned experiments such as interferometric detectors.

## INTRODUCTION

Recent studies, including strong coupling corrections to the string effective action, have provided the pre-Big Bang scenario [1,2] with a number of promising models for the transition from the growing to the decreasing curvature regime [3–6]. However, a crucial question remains: is it possible to test this scenario and, in particular, to distinguish it from other candidate models which attempt to describe the very early universe?

The answer to this essential question is in principle in the affirmative, since the pre-Big Bang scenario incorporates a dilaton field coupled to the metric, which modifies the evolution of tensor perturbations. It is well known, indeed, that the transition from an inflationary period to the FRW radiation-dominated era is associated with the production of a background of relic gravitational waves. Such primordial signals decoupled from matter soon after the Planck era and, unlike electromagnetic radiation which underwent a complicated history until recombination, gravitons have been transmitted scarcely without interaction down to our present epoch. As a consequence, their present spectrum should be a faithful portrait of the very early universe, thus opening a window for the observation of processes occurring near the Planck scale, and for discriminating among various theories of high-energy models of cosmology and of unified interactions. With respect to the standard inflationary scenario, the amplification of tensor perturbations in the pre-Big Bang scenario is strongly enhanced in the limit of large frequencies [7,8], and the amplitude of the spectrum is normalised so as to match the string scale at the high-frequency end point of the spectrum [9,10]. As a result, it has been claimed [2,9] that the produced background of gravitational waves could be detected in the near future by various experiments such as the second (planned) generation of interferometric detectors [11,12].

Up to now, the above extended studies of tensor perturbations have been mostly performed in the context of the lowest-order effective action of string theory. However, the singular behaviour of the tree-level background cosmological solutions implies that such studies are a priori inadequate to describe the high-frequency branch of the spectrum. Indeed short-scale modes are expected to leave the Hubble radius during the high-curvature regime which has to remain bounded from above. This is a major problem for the prediction of the amplification of these high-frequency modes, the ones to be mostly detected (if at all).

The aim of this contribution<sup>1</sup> is to discuss the evolution and the spectral distribution of primordial gravitons in the context of non-singular cosmological backgrounds, by taking into account (in the perturbation equation) the full contribution of those corrections that are responsible for the regularisation of the background solution.

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# NON-SINGULAR BACKGROUNDS

In the pre-Big bang framework, our FRW-universe is presently understood to result from the collision of plane waves [13] emerging from the trivial string vacuum in the asymptotic past [14]. From this epoch onwards, the evolution of the universe, driven by the kinetic energy of the dilaton, undergoes an inflationary expansion with growing coupling and curvature. Such an evolution is usually derived by the 4-dimensional low energy effective action of string theory:

$$S_{eff} = \frac{1}{16\pi\alpha'} \int d^4x \sqrt{-g} \left\{ e^{-\phi} \left[ R + \phi_{,\mu} \phi^{,\mu} \right] + \mathcal{L}_c \right\}. \quad (1)$$

When the curvature reaches a maximal scale  $|H| \sim \mathcal{O}(\lambda_S^{-1})$ , where  $\lambda_S \sim \sqrt{\alpha'}$ , the universe is expected to be smoothly connected to the FRW regime, with a constant dilaton field. The most promising approach to the graceful exit problem [15] suggests a cure to the curvature and dilaton singularities by adding higher-order corrections to the string effective action (see for instance [16–18, 3, 4, 6]), whose sources are twofold: tree-level  $\alpha'$  corrections, resulting from the string tension expansion of the effective action and loop corrections, arising from the more conventional loop expansion in powers of the string coupling  $g_s = e^{\phi/2}$ . The tree-level  $\alpha'$  correction we will be considering (see [6] for a detailed analysis) includes the most general form for a correction to the string action up to fourth-order in derivatives [19]:

$$\mathcal{L}_{\alpha'} = \alpha' \lambda e^{-\phi} \left\{ c_1 R_{GB}^2 + c_2 \left[ R^{\mu\nu} - g^{\mu\nu} R \right] \phi_{,\mu} \phi_{,\nu} + c_3 \square \phi \phi^{,\mu} \phi_{,\mu} - c_4 (\phi^{,\mu} \phi_{,\mu})^2 \right\}, \quad (2)$$

where  $R_{GB}^2 \equiv R^{\mu\nu\rho\sigma} R_{\mu\nu\rho\sigma} - 4R^{\mu\nu} R_{\mu\nu} + R^2$  is the well-know Gauss-Bonnet combination ensuring the field equations will remain second order in the fields. In fixing the coefficients  $c_i$ 's, we require that the full action reproduces the string scattering amplitude, and thus impose the constraints  $2c_3 = -[c_2 + 2(c_1 + c_4)]$  and  $c_1 = -1$ , working in units  $\alpha' = 1$  [19]. The parameter  $\lambda$  allows us to move between different string theories and later we will set  $\lambda_0 = -1/4$  to agree with previous studies of the Heterotic string [18]. Another conventional expansion of string theory relies on the string coupling parameter  $g_s$ . There is as yet no definitive calculation of the full loop expansion of string theory, and we are left to speculate on plausible terms that will eventually make up the loop corrections. Multiplying each term of the tree-level  $\alpha'$  correction by a suitable power of the string coupling is the approach we will be considering here and has already met with some success [3, 5, 6]. Since the quantum loop corrections are not formally derived from a string loop expansion, we shall allow different coefficients  $d_i$  ( $e_i$ ) at one-loop in the string coupling (two-loop respectively), which are not necessarily subject to the previous constraints. The effective lagrangian density  $\mathcal{L}_c$  in Eq. (1) is thus a sum of the tree-level  $\alpha'$  and loop corrections,  $\mathcal{L}_c = \mathcal{L}_{\alpha'} + \mathcal{L}_q$  and in the present case takes the form

$$\mathcal{L}_c = \mathcal{L}_{\alpha'}(c_i) + A e^{\phi} \mathcal{L}_{\alpha'}(d_i) + B e^{2\phi} \mathcal{L}_{\alpha'}(e_i), \quad (3)$$

where  $\mathcal{L}_{\alpha'}$  is given by Eq. (2), with the extra constant parameters  $A$  and  $B$  actually controlling the onset of the loop corrections. Numerous combinations of these corrections regularise the evolution of the background, an example of which is pictured in Fig. 1.  $A$  and  $B$  are typically of order unity and have oppositive signs [6]. Thus, we can use these non-singular solutions to extend different studies based on the tree-level action and probe the high-frequency part of the spectrum of relic gravitons.

## RELIC GRAVITONS

We shall now focus on the generation of gravitational waves arising from linearised tensor perturbations. Although several mechanisms may contribute to the generation of gravitational waves from the initial vacuum state of string cosmology (dynamic dimensional reduction [20], time-dependence of the dilaton field [8]), we shall only consider here the usual contribution arising from the accelerated expansion of the external three-dimensional space. As already stressed in a number of papers [2, 7, 8], the production of high-frequency gravitons is strongly enhanced compared to that of the standard inflationary scenario. Following pionnering work in [21], we perturb the full action generated by Eq. (3) around a homogeneous and isotropic solution for the scale factor  $a(\eta)$  and the dilaton  $\phi(\eta)$ , where  $\eta$  is conformal time. In Fourier space and for each normal mode of tensor oscillations of our gravi-dilaton background  $\psi_k$ , we can rewrite the linearised wave equation in terms of

the eigenstates of the Laplace operator,  $\nabla^2 \psi_k = -k^2 \psi_k$ . In so doing, we obtain a generalised wave equation for the tensor perturbation:

$$\psi_k'' + \left\{ k^2 [1 + c(\eta)] - V(\eta) \right\} \psi_k = 0. \quad (4)$$

Here a prime denotes differentiating with respect to conformal time. We have defined the effective frequency shift  $c(\eta) \equiv (y/z)^2 - 1$  and the background quantities including the tree-level  $\alpha'$  and loop corrections are:

$$y^2(\eta) = e^{-\phi} \left[ a^2 + \{\oplus\}_0 + A\{\oplus\}_1 + B\{\oplus\}_2 \right], \quad (5)$$

$$z^2(\eta) = e^{-\phi} \left[ a^2 + \{\otimes\}_0 + A\{\otimes\}_1 + B\{\otimes\}_2 \right], \quad (6)$$

and

$$\{\oplus\}_n = \alpha' \lambda e^{n\phi} \left\{ 4c_1(n-1)^2 \phi'^2 + \frac{1}{2} c_2 \phi'^2 + 4c_1(n-1) \left( \phi'' - \frac{a'\phi'}{a} \right) \right\}, \quad (7)$$

$$\{\otimes\}_n = \alpha' \lambda e^{n\phi} \left\{ 4c_1(n-1) \frac{a'\phi'}{a} - \frac{1}{2} c_2 \phi'^2 \right\}. \quad (8)$$

The evolution equation for the perturbation Eq. (4) is the main result of this study, since it encodes the full contribution (through Eq. (5)–Eq. (8)) of those corrections we used to regularise the background evolution. We stress that in the limit of small coupling and low curvature ( $\alpha' \rightarrow 0$ ) we recover the usual Schrödinger-like wave equation since  $y, z \rightarrow a e^{-\phi/2}$  and thus  $c(\eta) \rightarrow 0$ . Indeed, describing an extremely weak coupling and low curvature regime, the tree-level solutions of the pre-Big Bang scenario are fully adequate in the asymptotic past to describe the gravi-dilaton background the metric perturbations are emerging from. This allows us to normalise our perturbation to positive frequency modes only,  $\psi_k(\eta) \sim \frac{1}{\sqrt{2k}} e^{-ik\eta}$  for  $\eta \rightarrow -\infty$ .

[width=5cm,height=4cm]Hubble.eps [width=5cm,height=4cm]aH.eps [width=5cm,height=4cm]spectre.eps

**FIGURE 1.** The left figure shows a non-singular evolution for the Hubble parameter  $H = \dot{a}/a$  and for  $\dot{\phi}/3$  as a function of the number of e-folds,  $N = \ln a$ . The centered figure shows the evolution of  $aH/\text{Max}(aH)$  as function of  $N$ . The pre-Big Bang phase takes place approximatively for  $-\infty < N \leq -3$ . This inflationary period is followed after a short transition by a string phase with nearly constant Hubble parameter and linearly growing (in cosmic time) dilaton for  $2 \leq N \leq 9$ . After a succesfull exit sourced by corrections in the string coupling, the background evolution enters the FRW radiation-dominated phase at  $N \simeq 16$ . This clearly suggests that comoving modes leaving the Hubble radius during the string phase ( $10^{-6} \leq k/\text{Max}(aH) \leq 10^{-2}$ ) will lead to a smaller slope in the spectrum of relic gravitons. The right figure shows the corresponding spectral distribution as function of logarithmic interval of frequency,  $d\rho_k/d\ln(k)$  in units  $k/k_{max}$ . The upper dashed line corresponds to the cubic spectrum (pertinent in the small frequency limit) resulting from the dilaton-driven epoch.

The differences between the tree-level wave equation and its generalisation Eq. (4) are expected to arise when the curvature scale becomes large in string units. As a consequence, the low frequency branch of the dimensionless spectral distribution of relic gravitons remains unaffected by these corrections, and is characterised by a cubic slope [2,7,8]. However, new features due to the corrections arise in the large frequency limit. First, a given comoving wavenumber  $k$  becomes time-dependent, as already observed for a special case in [21]. Second, the effective potential is strongly reinforced. As a consequence, such corrections induce an additional amplification for comoving modes leaving the horizon during the high-curvature transition. Figure 1 pictures both the evolution of the background quantities and the resulting spectral distribution for the relic gravitons. Of more pertinence is the correspondance between the slope  $\xi$  of the string branch of the relic gravitons spectrum and the choice of the coefficients for the tree-level  $\alpha'$  corrections. Preliminary results [22] suggest  $1 \leq \xi \leq 3$ , which could have considerable implications for the detection of such a primordial signal. Indeed, in [10], the authors argue that the frequency peak is typically of order  $\Omega_{gw}(\omega) \simeq 10^{-6}$  for a maximal amplified frequency  $\omega \simeq 10^{11}$  [Hz]. Saturating this high-frequency end point, and regardless of the duration of the string phase with constant Hubble parameter, the spectrum of relic gravitons from a pre-Big Bang phase could be at most of  $\mathcal{O}(10^{-15})$  at  $\omega \simeq 10^2$  [Hz]. The energy density stored in the cosmological gravitational waves  $\Omega_{gw}(\omega)$  is thus far below the sensitivity of the second (planned) generation of spatial detectors!

## CONCLUSION

In the context of the pre-Big Bang scenario of string cosmology, we have derived a modified tensor perturbation equation including those corrections we use to regularise the background evolutions. Such modifications, we believe are generic, and lead to new features in the string branch of the spectrum of primordial gravitational waves. Although the tree-level wave equation is adequate for a first estimate of the production of the gravitons during the pre-Big Bang phase, this work supports the idea that a full treatment is required when dealing with the string branch of the spectrum. Preliminary results suggest that, independent of the choice of coefficients in the higher-order corrections, the slope of the string branch of the spectrum is steeper than first expected. As a consequence, the cosmological background of relic gravitons may not match with the frequency range probed by (second) planned experiments.

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